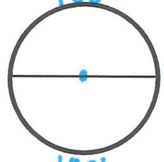
Circles

Parts and Properties of a Circle

Diameter

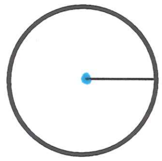
180°



180°

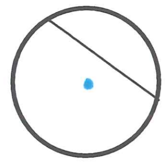
cuts the circle in half

Radius



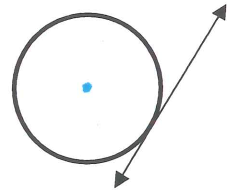
half of the diameter

Chord



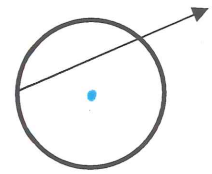
segment that joins 2 points on a circle

Tangent



a line outside of the circle that intersects in exactly 1 point

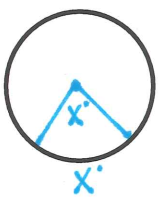
Secant



an extended chord (line or ray)

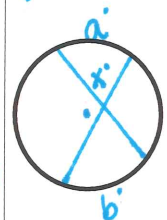
Angle and Arc Relationships *(There are 360° in a circle!)*

Central ∡ = intercepted arc



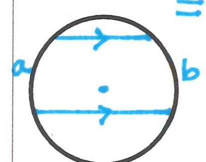
Inscribed ∡ = $\frac{1}{2}$ of the intercepted arc

∡ formed by 2 chords



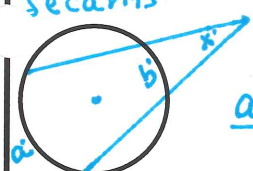
$x = \frac{a+b}{2}$

∥ chords intercept ≅ arcs



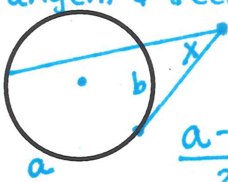
$a = b$

∡ formed by 2 secants



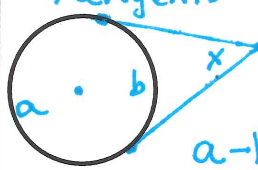
$\frac{a-b}{2} = x$

∡ formed by tangent & secant



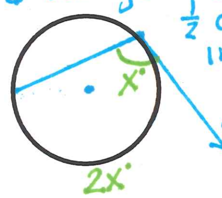
$\frac{a-b}{2} = x$

∡ formed by 2 tangents



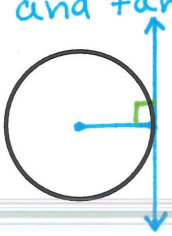
$\frac{a-b}{2} = x$

∡ formed by chord & tangent = $\frac{1}{2}$ of intercepted arc

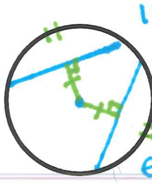


Length Relationships

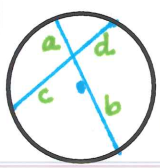
Radius (or diameter) and tangent are ⊥



≅ chords intercept ≅ arcs and are equidistant from center

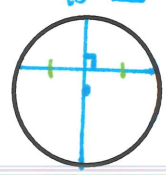


Intersecting chords

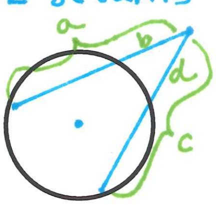


$a \cdot b = c \cdot d$

A diameter that is ⊥ to a chord, bisects the chord

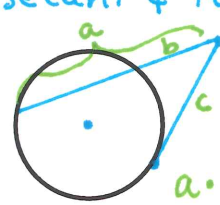


2 secants



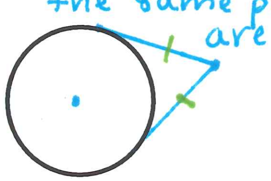
$a \cdot b = c \cdot d$

secant & tangent

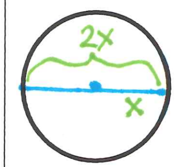


$a \cdot b = c^2$

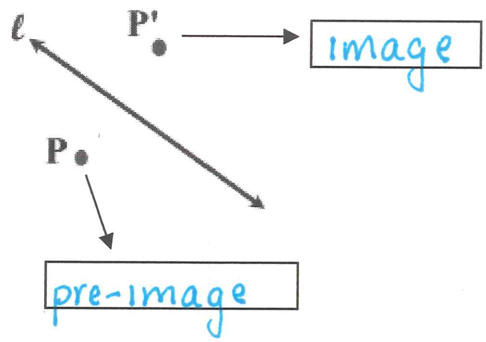
2 tangents from the same point are ≅



Radius is half of the diameter

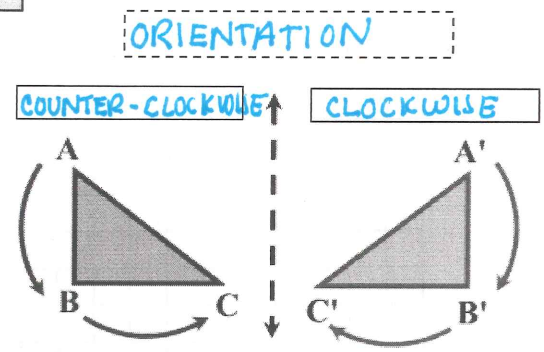


Transformations



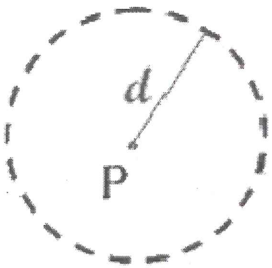
ISOMETRY

A transformation that preserves length

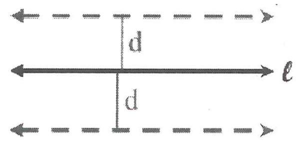


Name of Transformations	Properties	Example	What is preserved?	Is it an isometry? (Direct/Opposite)
Translation	slides an object a set distance in a given direction	$T_{2,-3}(x,y) =$ $(x,y) \rightarrow (x+2, y-3)$	•length •orientation •∡ measures	Yes! (Direct)
Reflection	flips an object over a point or line	$r_{y\text{-axis}}(x,y) =$ $(x,y) \rightarrow (-x,y)$ $r_{y=x}(x,y) =$ $(x,y) \rightarrow (y,x)$	•length •∡ measures	Yes! (opposite)
Rotation	turns an object a set # of degrees	$R_{90^\circ}(x,y) =$ $(x,y) \rightarrow (-y,x)$	•length •∡ measures •orientation	Yes! (Direct)
Dilation	enlarges or shrinks an object by a set #	$D_2(x,y) \rightarrow$ $(x,y) \rightarrow (2x,2y)$	•∡ measures •orientation	No!

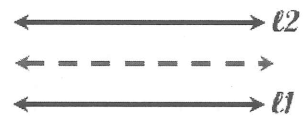
LOCUS



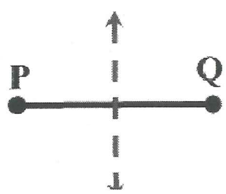
The locus of points at a fixed distance, ***d***, from point ***P*** is a circle with the given point ***P***as its center and ***d*** as its radius.



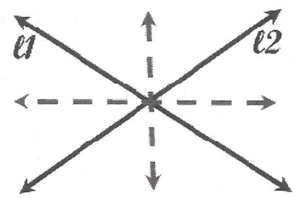
The locus of points at a fixed distance, ***d***, from a line, ℓ, is a pair of parallel lines ***d***distance from $\ell$and on either side of ℓ.



The locus of points equidistant from two points,***P*** and ***Q***, is the perpendicular bisector of the line segment determined by the two points.



The locus of points equidistant from two parallel lines, ℓ_1 and ℓ_2 , is a line parallel to both ℓ_1 and $\ell_2$and midway between them.



The locus of points equidistant from two intersecting lines, ℓ_1 and ℓ_2, is a pair of bisectors that bisect the angles formed by ℓ_1 and ℓ_2 .

Steps for Solving Locus Problems

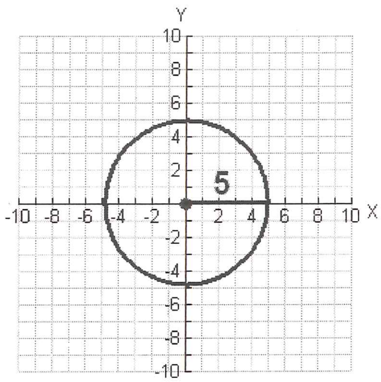
1. Draw a diagram showing the given lines and points
2. Read carefully to determine the needed condition(s).
3. Locate 1 point that satisfies the needed condition and plot it on your diagram. Repeat this process until you notice a pattern.
4. Connect your points with a dashed line to indicate the locus.
5. Describe the locus in words (circle, ll lines, etc.)
6. If 2 conditions exist, repeat the steps on the same diagram and identify the points of intersection.

Equations of Circles

Circle with Center at Origin (0,0)

$$x^2 + y^2 = r^2$$

where the center is (0,0) and the radius is r.

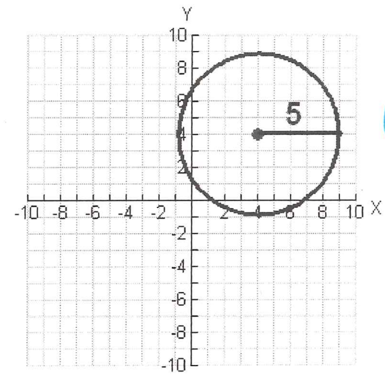


$x^2 + y^2 = 25$

Circle with Center at Point (h,k)

$$(x-h)^2 + (y-k)^2 = r^2$$

where the center is (h,k) and the radius is r

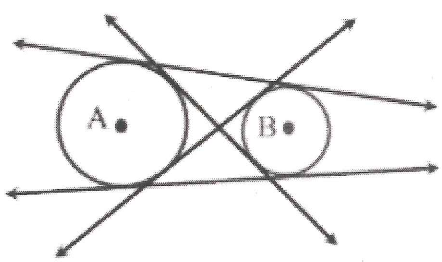


$(x-4)^2 + (y-4)^2 = 25$

Common Tangents

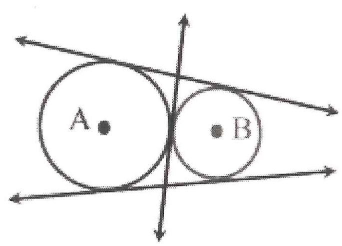
Common tangents are lines or segments that are tangent to more than one circle at the same time.

4 Common Tangents
(2 completely separate circles)



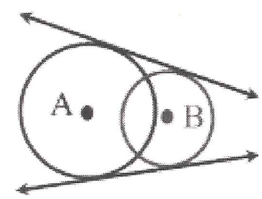
2 external tangents
2 internal tangents

3 Common Tangents
(2 externally tangent circles)



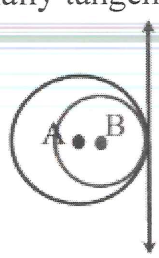
2 external tangents
1 internal tangent

2 Common Tangents
(2 overlapping circles)



2 external tangents
0 internal tangents

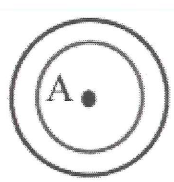
1 Common Tangent
(2 internally tangent circles)



1 external tangent
0 internal tangents

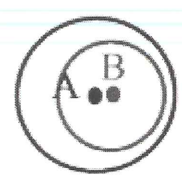
0 Common Tangents

(2 concentric circles)
Concentric circles are circles with the same center.



0 external tangents
0 internal tangents

(one circle floating inside the other, without touching)



0 external tangents
0 internal tangents