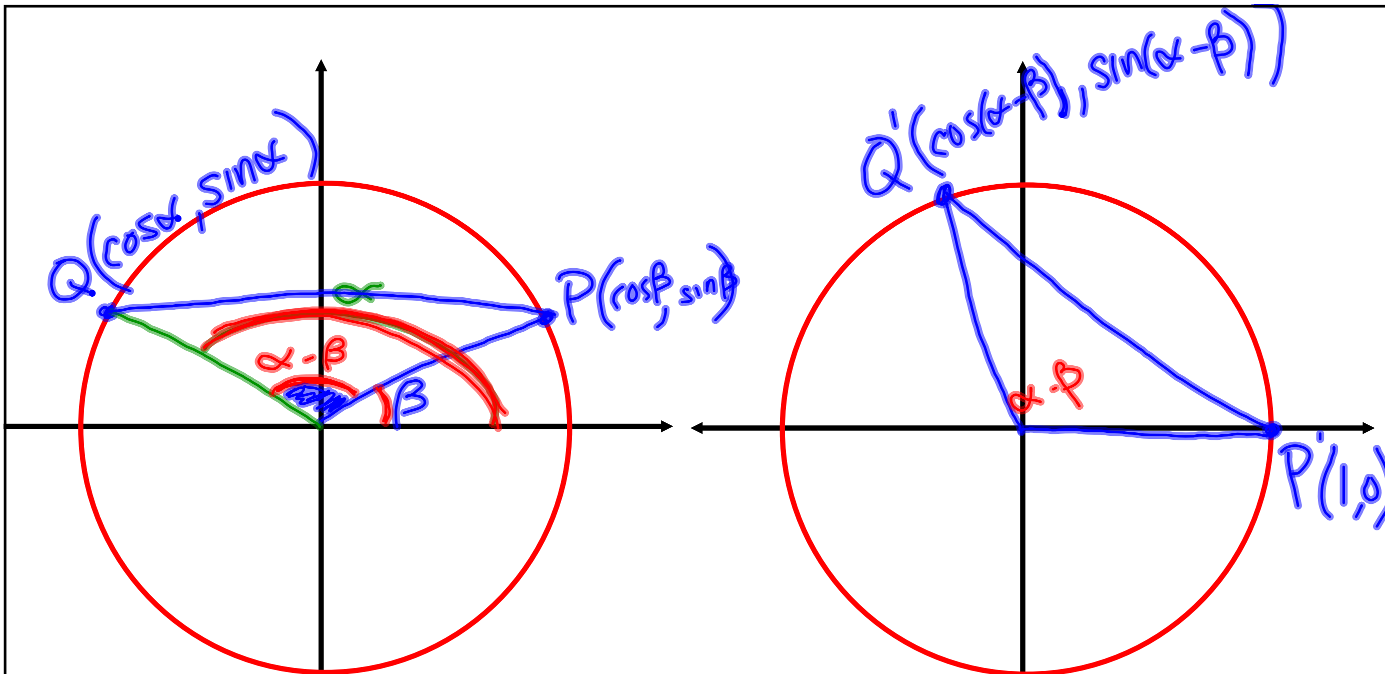
The Cosine of the Difference of Two Angles

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$



$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$P(\cos\beta, \sin\beta)$

$Q(\cos\alpha, \sin\alpha)$

$$\sqrt{(\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin\alpha)^2}$$

$$\sqrt{\cos^2\beta - 2\cos\beta\cos\alpha + \cos^2\alpha + \sin^2\beta - 2\sin\beta\sin\alpha + \sin^2\alpha}$$

$P'(1,0)$

$Q'(\cos(\alpha-\beta), \sin(\alpha-\beta))$

$$\sqrt{\cos^2\beta + \sin^2\beta + \cos^2\alpha + \sin^2\alpha - 2\cos\beta\cos\alpha - 2\sin\beta\sin\alpha}$$

$$\sqrt{1 + 1 - 2\cos\beta\cos\alpha - 2\sin\beta\sin\alpha}$$

$$\sqrt{2 - 2\cos\beta\cos\alpha - 2\sin\beta\sin\alpha}$$

$$\sqrt{(1-\cos(\alpha-\beta))^2 + (0-\sin(\alpha-\beta))^2}$$

$$\sqrt{1-2\cos(\alpha-\beta)+\cos^2(\alpha-\beta)+\sin^2(\alpha-\beta)}$$

$$\sqrt{1-2\cos(\alpha-\beta)+1}$$

$$\sqrt{2-2\cos(\alpha-\beta)}$$

$$\left(\sqrt{2-2\cos(\alpha-\beta)}\right)^2 = \left(\sqrt{2-2\cos\beta\cos\alpha-2\sin\beta\sin\alpha}\right)^2$$

$$\cancel{2}-2\cos(\alpha-\beta) = \cancel{2}-2\cos\beta\cos\alpha-2\sin\beta\sin\alpha$$
$$-\cancel{2} \qquad\qquad -\cancel{2}$$

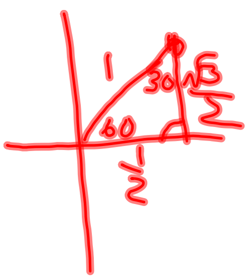
$$\frac{-\cancel{2}\cos(\alpha-\beta)}{-\cancel{2}} = \frac{-\cancel{2}\cos\beta\cos\alpha \overset{+}{-} \cancel{2}\sin\beta\sin\alpha}{-\cancel{2}}$$

$$\cos(\alpha-\beta) = \cos\beta\cos\alpha + \sin\beta\sin\alpha$$

Find the exact value of cos15°.

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\cos(60-45)=\cos 60\cos 45+\sin 60\sin 45$$



$$=\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$$

$$=\frac{\sqrt{2}+\sqrt{6}}{4}$$

Find the exact value of cos80°cos20° + sin80°sin20°.

$\cos(80-20)$

$\cos(60)$

$\frac{1}{2}$

Verify the identity: $\dfrac{\cos(\alpha-\beta)}{\sin\alpha\cos\beta}=\cot\alpha+\tan\beta$