Sum and Difference Formulas for Cosines and Sines

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

How can we use the cosine of a difference of two angles to verify:

$$\cos(\alpha+\beta) = \cos(\alpha-(-\beta))$$

$$= \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta)$$

$$= \cos\alpha\cos\beta + \sin\alpha(-\sin(\beta))$$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

How can we use the cosine of a difference of two angles to verify:

$\sin\theta = \cos(90-\theta)$

(Hint: Use cofunction identity.)

$\sin(\alpha+\beta) = \cos(90-(\alpha+\beta))$

$= \cos((90-\alpha)-\beta)$

$= \cos(90-\alpha)\cos(-\beta) + \sin(90-\alpha)\sin(-\beta)$

$= \sin\alpha\cos\beta + \cos\alpha\cdot(-\sin\beta)$

$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$

How can use the sine of the sum of two angles to verify:

$$\sin(\alpha - \beta) = \sin(\alpha + (-\beta))$$

$$= \sin\alpha\cos(-\beta) - \cos\alpha\sin(-\beta)$$

$$= \sin\alpha\cos\beta - \cos\alpha(-\sin\beta)$$

$$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Find the exact value of cos(45° - 30°).

$\cos 15°$

$= \cos 45 \cos 30 + \sin 45 \sin 30$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

Find the exact value of $\sin\frac{7\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= \sin(\overset{\alpha}{45} + \overset{\beta}{60})$

$= \sin 45 \cos 60 - \cos 45 \sin 60$

$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

April 28, 2014